**Assignment 2**

**Group 3**

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**1. Option Pricing in the Binomial Model**

1. We can calculate the stock price at each node with binomial tree model:

Then, we have the binomial tree of stock price:

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We then need to calculate the probability of q and (1-q):

**To calculate the American call option price:**

we can calculate the payoff at period t by:

And we work backwards to get the option payoff at each node:

Finally, at each node, we need to compare:

Therefore, we get the binomial tree of American call option. The call option price at time 0 is equal to 2.68

Table

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1. **To calculate the American put option price:**

we can calculate the payoff at period t by:

And we work backwards to get the option payoff at each node:

Finally, at each node, we need to compare:

Therefore, we get the binomial tree of American put option. The put option price at time 0 is equal to 12.23

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1. It is optimal to early exercise the American put option, if at some nodes:

For American put option in part (b), we can early exercise it.

1. Based on the logic mentioned in part (c), we can get the optimal earliest period to exercise the American put option is at period 7.

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1. Since American options allow early exercise, put-call parity will not hold for American options unless they are held to expiration. Early exercise will result in a departure in the present values of the two portfolios.

**2. Pricing Futures Contracts**

Since a future contract always worth 0, we have

Because R is constant, we have

Therefore,

Since at is known, we have

Therefore, for :

According to tower property, for any variable x and :

Therefore,

Since for a future contract,

Hence,

Since there is no dividend paid,

So, we have

Since and are both constant,

**3. Convergence of Binomial Model Option Prices to Black-Scholes Prices**

1. Compute the prices of European call options using Black-Scholes model

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1. Compute the prices of European call options using binomial model

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1. As the number of period n increases, the option price calculated by the binomial model will gradually move closer to the option price obtained via Black-Scholes model. As you can see in the graph below, the option price when n=10 is furthest from the Black-Scholes model's option price, while the option price is very similar to that of from Black-Scholes when n=1000. That is to say, the option price calculated by binomial model will converge to the Black-Scholes option price.

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Chart

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**4. Dynamic Hedging in the Black-Scholes Model**